Large Mixing and CP Violation in Neutrino Oscillations

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I introduce a simple phenomenological model of lepton flavor mixing and CP violation based on the flavor democracy of charged leptons and the mass degeneracy of neutrinos. The nearly bi-maximal mixing pattern, which can interpret current data on atmospheric and solar neutrino oscillations, emerges naturally from this model. The rephasing-invariant strength of CP or T violation amounts to about one percent and could be measured in the long-baseline neutrino experiments. The similarity and difference between lepton and quark flavor mixing phenomena are also discussed.

1 In the standard electroweak model neutrinos are assumed to be the massless Weyl particles. This assumption, which has no conflict with all direct-mass-search experiments [1], is not guaranteed by any fundamental principle of particle physics. Indeed most extensions of the standard model, such as the grand unified theories of quarks and leptons, allow the existence of massive neutrinos. If the masses of three active neutrinos (ν_e, ν_μ and ν_τ) are nonvanishing, why are they so small in comparison with the masses of charged leptons or quarks? For the time being this question remains open, although a lot of theoretical speculations towards a definite answer have been made. The smallness of neutrino masses is perhaps attributed to the fact that neutrinos are electrically neutral fermions, or more exactly, to the Majorana feature of neutrino fields.

Recent observation of the atmospheric and solar neutrino anomalies, particularly that in the Super-Kamiokande experiment, has provided strong evidence that neutrinos are massive and lepton flavors are mixed. Analyses of the atmospheric neutrino deficit in the framework of either two or three lepton flavors favor $\nu_\mu \rightarrow \nu_\tau$ as the dominant oscillation mode [2,3] and yield the following mass-squared difference and mixing factor at the 90% confidence level [2]:

$$\Delta m^2_{\rm atm} \sim 10^{-3} \ {\rm eV}^2 \ , \qquad \sin^2 2\theta_{\rm atm} > 0.9 \ . \qquad (1)$$

As for the solar neutrino anomaly, the hypothesis that solar ν_e neutrinos change to ν_μ neutrinos during their travel to the earth through the long-wavelength vacuum oscillation with the parameters

$$\Delta m^2_{\rm sun} \sim 10^{-10} \ {\rm eV}^2 \ , \qquad \sin^2 2\theta_{\rm sun} \approx 1 \ , \qquad (2)$$

can provide a consistent explanation of all existing solar neutrino data [4]. Alternatively the large-angle MSW (Mikheyev, Smirnov, and Wolfenstein) solution, i.e., the matter-enhanced $\nu_e \rightarrow \nu_\mu$ oscillation with the parameters

$$\Delta m^2_{\rm sun} \sim 10^{-5} \ {\rm eV}^2 \ , \qquad \sin^2 2\theta_{\rm sun} \sim 1 \ , \qquad (3)$$

seems also favored by current data [5]. To distinguish between the MSW and vacuum solutions to the solar neutrino problem is a challenging task of the next-round solar neutrino experiments.

Current data indicate that solar and atmospheric neutrino oscillations are approximately decoupled from each other. Each of them is dominated by a single mass scale, i.e.,

$$\begin{aligned} \Delta m^2_{\rm sun} &= \left|\Delta m^2_{21}\right| = \left|m^2_2 - m^2_1\right| \ , \\ \Delta m^2_{\rm atm} &= \left|\Delta m^2_{32}\right| = \left|m^2_3 - m^2_2\right| \ , \end{aligned} \qquad (4)$$

and $\Delta m^2_{31} \approx \Delta m^2_{32}$ in the scheme of three lepton flavors. In addition, the ν_3-component in ν_e is rather small; i.e., the V_{e3} element of the lepton flavor mixing matrix V, which links the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$, is suppressed in magnitude. Note, however, that the hierarchy of Δm^2_{21} and Δm^2_{32} (or Δm^2_{31}) cannot give clear

*Talk given at the Sixth Topical Seminar on Neutrino and Astroparticle Physics, San Miniato, Italy, May 1999

information about the absolute values or the relative magnitude of three neutrino masses. For example, either the strongly hierarchical neutrino mass spectrum ($m_1 \ll m_2 \ll m_3$) or the nearly degenerate one ($m_1 \approx m_2 \approx m_3$) is allowed to reproduce the "observed" mass gap between $\Delta m^2_{\rm sun}$ and $\Delta m^2_{\rm atm}$.

In the presence of flavor mixing among three different fermion families, CP violation is generally expected to appear. This is the case for quarks, and there is no reason why the same phenomenon does not manifest itself in the lepton sector [6]. The strength of CP violation in neutrino oscillations, no matter whether neutrinos are of the Dirac or Majorana type, depends only upon a universal (rephasing-invariant) parameter $\mathcal{J}$, which is defined through

$$\mathrm{Im}\left(V_{il}V_{jm}V^*_{im}V^*_{jl}\right) \; = \; \mathcal{J}\sum_{k,n}\epsilon_{ijk}\epsilon_{lmn} \; . \tag{5}$$

The asymmetry between the probabilities of two CP-conjugate neutrino transitions, due to the CPT invariance and the unitarity of V, are uniquely given as

$$\begin{aligned} \mathcal{A}_{CP} &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \\ &= -16\mathcal{J}\sin F_{12}\sin F_{23}\sin F_{31} \end{aligned} \tag{6}$$

with $(\alpha,\beta) = (e,\mu)$, (μ,τ) or (τ,e), $F_{ij} = 1.27\Delta m^2_{ij}L/E$ and $\Delta m^2_{ij} = m^2_i - m^2_j$, in which L is the distance between the neutrino source and the detector (in unit of Km) and E denotes the neutrino beam energy (in unit of GeV). The T-violating asymmetries can be obtained in a similar way [7]:

$$\begin{aligned} \mathcal{A}_{T} &= P(\nu_\alpha \rightarrow \nu_\beta) - P(\nu_\beta \rightarrow \nu_\alpha) \\ &= -16\mathcal{J}\sin F_{12}\sin F_{23}\sin F_{31} \; , \\ \mathcal{A}'_{T} &= P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) - P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) \\ &= +16\mathcal{J}\sin F_{12}\sin F_{23}\sin F_{31} \; , \end{aligned} \tag{7}$$

where $(\alpha,\beta) = (e,\mu)$, (μ,τ) or (τ,e). These formulas show clearly that CP or T violation is a behavior of all three lepton families. In addition, the relationship $\mathcal{A}'_T = -\mathcal{A}_T$ indicates that the two T-violating measurables are odd functions of time [8]. A necessary condition for obtaining large (observable) CP or T violation is that the magnitude of $\mathcal{J}$ should be large enough. In view of the smallness of $|V_{e3}|$, one may conclude that the largeness of $|\mathcal{J}|$ requires the largeness of both $|V_{e2}|$ and $|V_{\mu3}|$. Therefore a mixing scheme which can accommodate the small-angle MSW solution to the solar neutrino problem (due to the smallness of $|V_{e2}|$) would not be able to give rise to large CP or T violation.

In the following I present a phenomenological model for lepton mass generation and CP violation within the framework of three lepton species (i.e., the LSND evidence for neutrino oscillations, which was not confirmed by the KARMEN experiment [9], is not taken into account). The basic idea, first pointed out by Fritzsch and me in 1996 [10] to get nearly bi-maximal lepton flavor mixing, is associated with the flavor democracy of charged leptons and the mass degeneracy of active neutrinos. We introduce a simple flavor symmetry breaking scheme for charged lepton and neutrino mass matrices, so as to generate two nearly bi-maximal flavor mixing angles and to interpret the approximate decoupling of solar and atmospheric neutrino oscillations. Large CP or T violation of order $|\mathcal{J}| \sim 1\%$ can naturally emerge in this scenario. Consequences on the upcoming long-baseline neutrino experiments, as well as the similarity and difference between lepton and quark mixing phenomena, will also be discussed.

2 Let me start with the symmetry limits of the charged lepton and neutrino mass matrices. In a specific basis of flavor space, in which charged leptons have the exact flavor democracy and neutrino masses are fully degenerate, the mass matrices can be written as [10,11]

$$\begin{aligned} M^{(0)}_l &= \frac{c_l}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \; , \\ M^{(0)}_\nu &= c_\nu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; , \end{aligned} \tag{8}$$

where $c_l = m_\tau$ and $c_\nu = m_0$ measure the corresponding mass scales. If the three neutrinos are of the Majorana type, $M^{(0)}_\nu$ could take a more gen-

eral form $M_\nu^{(0)} P_\nu$ with $P_\nu = \mathrm{Diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$. As the Majorana phase matrix P_ν has no effect on the flavor mixing and CP-violating observables in neutrino oscillations, it will be neglected in the subsequent discussions. Clearly $M_\nu^{(0)}$ exhibits an S(3) symmetry, while $M_l^{(0)}$ an $S(3)_{\rm L} \times S(3)_{\rm R}$ symmetry. In these limits $m_e = m_\mu = 0$, $m_1 = m_2 = m_3 = m_0$, and no flavor mixing is present.

A simple real diagonal breaking of the flavor democracy for $M_l^{(0)}$ and the mass degeneracy for $M_\nu^{(0)}$ may lead to instructive results for flavor mixing in neutrino oscillations [10–12]. To accommodate CP violation, however, complex perturbative terms are required [7]. Let me proceed with two different symmetry-breaking steps.

(a) Small real perturbations to the (3,3) elements of $M_l^{(0)}$ and $M_\nu^{(0)}$ are respectively introduced [13]:

$$\begin{aligned}
\Delta M_l^{(1)} &= \frac{c_l}{3} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_l \end{pmatrix} , \\
\Delta M_\nu^{(1)} &= c_\nu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_\nu \end{pmatrix} .
\end{aligned} \tag{9}$$

In this case the charged lepton mass matrix $M_l^{(1)} = M_l^{(0)} + \Delta M_l^{(1)}$ remains symmetric under an $S(2)_{\rm L} \times S(2)_{\rm R}$ transformation, and the neutrino mass matrix $M_\nu^{(1)} = M_\nu^{(0)} + \Delta M_\nu^{(1)}$ has an $S(2)$ symmetry. The muon becomes massive ($m_\mu \approx 2|\varepsilon_l| m_\tau/9$), and the mass eigenvalue m_3 is no more degenerate with m_1 and m_2 (i.e., $|m_3 - m_0| = m_0 |\varepsilon_\nu|$). After the diagonalization of $M_l^{(1)}$ and $M_\nu^{(1)}$, one finds that the 2nd and 3rd lepton families have a definite flavor mixing angle θ. We obtain $\tan\theta \approx -\sqrt{2}$ if the small correction of $O(m_\mu/m_\tau)$ is neglected. Then neutrino oscillations at the atmospheric scale may arise from $\nu_\mu \rightarrow \nu_\tau$ transitions with $\Delta m^2_{32} = \Delta m^2_{31} \approx 2 m_0 |\varepsilon_\nu|$. The corresponding mixing factor $\sin^2 2\theta \approx 8/9$ is in good agreement with current data.

(b) Small imaginary perturbations, which have the identical magnitude but the opposite signs, are introduced to the (2,2) and (1,1) elements of $M_l^{(1)}$. For $M_\nu^{(1)}$ the corresponding real perturbations are taken into account [7]:

$$\begin{aligned}
\Delta M_l^{(2)} &= \frac{c_l}{3} \begin{pmatrix} -i\delta_l & 0 & 0 \\ 0 & i\delta_l & 0 \\ 0 & 0 & 0 \end{pmatrix} , \\
\Delta M_\nu^{(2)} &= c_\nu \begin{pmatrix} -\delta_\nu & 0 & 0 \\ 0 & \delta_\nu & 0 \\ 0 & 0 & 0 \end{pmatrix} .
\end{aligned} \tag{10}$$

We obtaine $m_e \approx |\delta_l|^2 m_\tau^2/(27 m_\mu)$ and $m_1 = m_0(1-\delta_\nu)$, $m_2 = m_0(1+\delta_\nu)$. The diagonalization of $M_l^{(2)} = M_l^{(1)} + \Delta M_l^{(2)}$ and $M_\nu^{(2)} = M_\nu^{(1)} + \Delta M_\nu^{(2)}$ leads to a full 3×3 flavor mixing matrix, which links neutrino mass eigenstates (ν_1, ν_2, ν_3) to neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$ in the following manner:

$$V = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} + \Delta V \tag{11}$$

with

$$\Delta V = i\, \xi_V \sqrt{\frac{m_e}{m_\mu}} + \zeta_V \frac{m_\mu}{m_\tau} , \tag{12}$$

where

$$\begin{aligned}
\xi_V &= \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} , \\
\zeta_V &= \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{12}} & \frac{1}{\sqrt{3}} \end{pmatrix} .
\end{aligned} \tag{13}$$

Some consequences of this mixing scenario can be drawn as follows:

(1) The mixing pattern in Eq. (11), after neglecting small corrections from the charged lepton masses, is quite similar to that of the pseudoscalar mesons π^0, η and η' in QCD [14]. One may speculate whether such an analogy could be taken as a hint towards an underlying symmetry and its breaking, which are responsible for lepton mass generation and flavor mixing [15,16].

(2) The V_{e3} element, of magnitude

$$|V_{e3}| = \frac{2}{\sqrt{6}} \sqrt{\frac{m_e}{m_\mu}} , \tag{14}$$

is naturally suppressed in the symmetry breaking scheme outlined above. A similar feature appears in the 3×3 quark flavor mixing matrix, i.e., $|V_{ub}|$ is the smallest among the nine quark mixing elements [17]. Indeed the smallness of V_{e3} provides a necessary condition for the decoupling of solar and atmospheric neutrino oscillations, even though neutrino masses are nearly degenerate. The effect of small but nonvanishing V_{e3} can manifest itself in the long-baseline $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\tau$ transitions, as shown in Ref. [11].

(3) The flavor mixing between the 1st and 2nd lepton families and that between the 2nd and 3rd lepton families are nearly maximal [10]. This property, together with the natural smallness of V_{e3}, allows a satisfactory interpretation of the observed large mixing in atmospheric and solar neutrino oscillations. We obtain

$$\begin{aligned} \sin^2 2\theta_{\rm sun} &= 1 - \frac{4}{3}\frac{m_e}{m_\mu} \; , \\ \sin^2 2\theta_{\rm atm} &= \frac{8}{9} + \frac{8}{9}\frac{m_\mu}{m_\tau} \; , \end{aligned} \tag{15}$$

to a quite high degree of accuracy. Explicitly $\sin^2 2\theta_{\rm sun} \approx 0.99$ and $\sin^2 2\theta_{\rm atm} \approx 0.94$, favored by current data [2]. It is obvious that the model is fully consistent with the vacuum oscillation solution to the solar neutrino problem [4] and might also be able to incorporate the large-angle MSW solution [18].

(4) The nearly degenerate neutrino masses and nearly bi-maximal mixing angles in the present scenario make it possible to accommodate the hot dark matter of the universe in no conflict with the constraint from the neutrinoless double beta decay (denoted by $(\beta\beta)_{0\nu}$). The former requirement can easily be fulfilled, if one takes $m_i \sim 1-2$ eV (for $i = 1, 2, 3$). The effective mass term of the $(\beta\beta)_{0\nu}$ decay, in the presence of CP violation, is written as

$$\langle m \rangle = \sum_{i=1}^{3} \left(m_i U_{ei}^2 \right) \; , \tag{16}$$

where $U = V P_\nu$, and $P_\nu = {\rm Diag}\{1, e^{i\phi_1}, e^{i\phi_2}\}$ is a diagonal phase matrix of the Majorana nature. Taking $\phi_1 = \phi_2 = \pi/2$ for example, we arrive at

$$|\langle m \rangle| = \frac{2}{\sqrt{3}} \sqrt{\frac{m_e}{m_\mu}} \; m_i \; , \tag{17}$$

i.e., $|\langle m \rangle| \approx 0.08 m_i \leq 0.2$ eV, the latest experimental bound of the $(\beta\beta)_{0\nu}$ decay [19].

(5) The strength of CP violation in this scheme is given by [7]

$$\mathcal{J} \approx \frac{1}{3\sqrt{3}} \sqrt{\frac{m_e}{m_\mu}} \left(1 + \frac{1}{2}\frac{m_\mu}{m_\tau} \right) \; . \tag{18}$$

Explicitly we have $\mathcal{J} \approx 0.014$. The large magnitude of $\mathcal{J}$ for lepton mixing is remarkable, as the same quantity for quark mixing is only of order 10^{-5} [20]. In view of the approximate decoupling of solar and atmospheric neutrino oscillations, the CP- and T-violating asymmetries presented in Eqs. (6) and (7) can be simplified to the following form in a long-baseline neutrino experiment:

$$\mathcal{A}_{CP} = \mathcal{A}_T \approx 16\mathcal{J} \sin F_{12} \sin^2 F_{23} \; . \tag{19}$$

We see that the maximal magnitude of $\mathcal{A}_{CP}$ or $\mathcal{A}_T$ is able to reach $16\mathcal{J} \approx 0.2$, significant enough to be measured from the asymmetry between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ or that between $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_e \rightarrow \nu_\mu)$ in the long-baseline neutrino experiments with the condition $E/L \sim |\Delta m^2_{12}|$.

Of course the afore-mentioned requirement for the length of the baseline singles out the large-angle MSW solution, whose oscillation parameters are given in Eq. (3), among three possible solutions to the solar neutrino problem. If upcoming neutrino oscillation data turn out to rule out the consistency between our model and the large-angle MSW scenario, then it would be quite difficult to test the model itself from its prediction for large CP and T asymmetries in the realistic long-baseline experiments.

It is at this point worth mentioning that the diagonal non-hermitian perturbation to $M_l^{(0)}$ is not the only way to generate large CP violation in our model. Instead one may consider the off-diagonal non-hermitian perturbations

$$\Delta \tilde{M}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & 0 & i\delta_l \\ 0 & 0 & -i\delta_l \\ i\delta_l & -i\delta_l & 0 \end{pmatrix} \; ,$$

$$\Delta \hat{M}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} -i\delta_l & 0 & i\delta_l \\ 0 & i\delta & -i\delta_l \\ i\delta_l & -i\delta_l & 0 \end{pmatrix} ; \qquad (20)$$

or the off-diagonal hermitian perturbations

$$\Delta \mathbf{M}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & -i\delta_l & 0 \\ i\delta_l & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,$$
$$\Delta \tilde{\mathbf{M}}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & 0 & i\delta_l \\ 0 & 0 & -i\delta_l \\ -i\delta_l & i\delta_l & 0 \end{pmatrix} ,$$
$$\Delta \hat{\mathbf{M}}_l^{(2)} = \frac{c_l}{3} \begin{pmatrix} 0 & -i\delta_l & i\delta_l \\ i\delta & 0 & -i\delta_l \\ -i\delta_l & i\delta_l & 0 \end{pmatrix} , \qquad (21)$$

for the same purpose. Note that all the six perturbative mass matrices $\Delta M_l^{(2)}$, $\Delta \tilde{M}_l^{(2)}$, $\Delta \hat{M}_l^{(2)}$ and $\Delta \mathbf{M}_l^{(2)}$, $\Delta \tilde{\mathbf{M}}_l^{(2)}$, $\Delta \hat{\mathbf{M}}_l^{(2)}$ have a common feature: the (1,1) elements of their counterparts in the hierarchical basis all vanish [7]. This feature assures that the CP-violating effects, resulted from the above complex perturbations, are approximately independent of other details of the flavor symmetry breaking and have the identical strength to a high degree of accuracy. Indeed it is easy to check that the relevant charged lepton mass matrices, together with the neutrino mass matrix $M_\nu^{(2)} = M_\nu^{(0)} + \Delta M_\nu^{(1)} + \Delta M_\nu^{(2)}$, lead to the same flavor mixing pattern V as given in Eq. (11) [7]. Hence it is in practice difficult to distinguish one scenario from another. In our point of view, the simplicity of $M_l^{(2)}$ and its parallelism with $M_\nu^{(2)}$ might make it technically more natural to be derived from a yet unknown fundamental theory of lepton mixing and CP violation.

3 In the scheme of three lepton species I have introduced a simple phenomenological model for lepton mass generation, flavor mixing and CP violation. The model starts from the flavor democracy of charged leptons and the mass degeneracy of neutrinos. After the symmetry limits of both mass matrices are explicitly broken, we find that this scenario can naturally give rise to large flavor mixing and large CP or T violation. The condition for the approximate decoupling of atmospheric and solar neutrino oscillations (i.e., $|V_{e3}| \ll 1$) is also fulfilled.

The lepton mixing pattern, which arises from $m_e \ll m_\mu \ll m_\tau$ and $m_1 \approx m_2 \approx m_3$, seems to be "anomalously" different from the quark mixing pattern obtained from the fact $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$. However, their similarities do exist. To see this point clearly, let me parametrize the quark (Q) and lepton (L) flavor mixing matrices in an instructive way [21]:

$$V_{\rm Q} = R_{12}(\theta_{\rm u}, 0) R_{23}(\theta_{\rm Q}, -\phi_{\rm Q}) R_{12}^{\rm T}(\theta_{\rm d}, 0) ,$$
$$V_{\rm L} = R_{12}(\theta_l, 0) R_{23}(\theta_{\rm L}, -\phi_{\rm L}) R_{12}^{\rm T}(\theta_\nu, 0) , \qquad (22)$$

where the complex rotation matrices R_{12} and R_{23} are defined as

$$R_{12}(\theta, \phi) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix} ,$$
$$R_{23}(\theta, \phi) = \begin{pmatrix} e^{i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \qquad (23)$$

with $c \equiv \cos\theta$ and $s \equiv \sin\theta$. Note that the rotation sequence of $V_{\rm Q}$ or $V_{\rm L}$ is essentially the original Euler sequence with an additional CP-violating phase. The rotation angle θ_l (or θ_ν) mainly describes the mixing between e and μ leptons (or between ν_e and ν_μ neutrinos), and the rotation angle $\theta_{\rm u}$ (or $\theta_{\rm d}$) primarily describes the mixing between u and c quarks (or between d and s quarks). The rotation angle $\theta_{\rm Q}$ (or $\theta_{\rm L}$) is a combined effect arising from the mixing between the 2nd and 3rd families for quarks (or leptons). The phase parameters $\phi_{\rm Q}$ and $\phi_{\rm L}$ signal CP violation in flavor mixing (for neutrinos of the Majorana type, two additional CP-violating phases may enter but they are irrelevant for neutrino oscillations). Comparing Eqs. (11)–(13) with (22) we immediately arrive at

$$\tan\theta_l = \sqrt{\frac{m_e}{m_\mu}} , \qquad \tan\theta_\nu = \sqrt{\frac{m_1}{m_2}} . \qquad (24)$$

In contrast, a variety of quark mass matrices have predicted [20,22]

$$\tan\theta_{\rm u} = \sqrt{\frac{m_u}{m_c}} , \qquad \tan\theta_{\rm d} = \sqrt{\frac{m_d}{m_s}} . \qquad (25)$$

As one can see, the large mixing angle θ_ν (i.e., $\tan\theta_\nu \approx 1$ for almost degenerate m_1 and m_2)

is attributed to the near degeneracy of neutrino masses in our flavor symmetry breaking scheme. Explicitly three mixing angles of leptons take the values [7]

$$\theta_l \approx 4^\circ \; , \quad \theta_\nu \approx 45^\circ \; , \quad \theta_{\rm L} \approx 52^\circ \; ; \tag{26}$$

and those of quarks take the values [23]

$$\theta_{\rm u} \approx 5^\circ \; , \quad \theta_{\rm d} \approx 11^\circ \; , \quad \theta_{\rm Q} \approx 2^\circ \; . \tag{27}$$

Furthermore, the CP-violating phases $\phi_{\rm L}$ and $\phi_{\rm Q}$ are both close to a special value:

$$\phi_{\rm Q} \; \approx \; \phi_{\rm L} \; \approx \; 90^\circ \; . \tag{28}$$

Let me emphasize that the possibility $\phi_{\rm Q} \approx 90^\circ$ is favored by a variety of realistic models of quark mass matrices [20]. The result $\phi_{\rm L} \approx 90^\circ$ is a distinctive feature of our lepton mixing scenario, but to verify it in a model-independent way would be extremely difficult, if not impossible, in the future neutrino experiments. Indeed the questions, about how large the feasibility is and how much the cost will be to measure CP or T violation in neutrino oscillations [6–8,24], remain open.

We are expecting that more data from the Super-Kamiokande and other neutrino experiments could provide stringent tests of the model discussed here.

Acknowledgments This talk is based on the works in collaboration with H. Fritzsch. I am indebted to him for his constant encouragement. I would like to thank F.L. Navarria and the organizing committee of *San Miniato 1999* for partial financial support. I am grateful to Q.Y. Liu, W.G. Scott and Y.L. Wu for intensive and interesting discussions during the workshop.

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